

AD-A264 181



AR-008-257

THE WAKE SOFTWARE SUITE: USER'S GUIDE

(2)

M.A. CARROLL

MRL-GD-0050

JANUARY 1993

DTIC  
ELECTE  
MAY 13 1993  
S C D

APPROVED

FOR PUBLIC RELEASE



Commonwealth of Australia

93-10324



4508

MATERIALS RESEARCH LABORATORY

DSTO

93 5 11 09 0

# *The Wake Software Suite – A Program to Predict the Internal Waves Generated by a Moving Subsurface Body in a Density Stratified Ocean: User's Guide*

M.A. Carroll

MRL General Document  
MRL-GD-0050

## *Abstract*

*This report provides details of the requirements needed to run the computer program WAKE. WAKE was developed during the period February-March 1992 as a result of a research project which was undertaken within DSTO Salisbury. WAKE calculates the internal wave velocity field generated by a moving subsurface body in a density stratified ocean.*

DSTO MATERIALS RESEARCH LABORATORY

**DTIC QUALITY INSPECTED 1**

Accession For	
NTIS CRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
A-1	

*Published on behalf of*

*M.A. Carroll  
EBOR Computing*

*by*

*Materials Research Laboratory  
Cordite Avenue, Maribyrnong  
Victoria, 3032 Australia*

*Telephone: (03) 246 8111*

*Fax: (03) 246 8999*

*© Commonwealth of Australia 1993*

*AR No. 008-257*

APPROVED FOR PUBLIC RELEASE

**THE WAKE SOFTWARE SUITE - A PROGRAM TO PREDICT THE  
INTERNAL WAVES GENERATED BY A MOVING SUBSURFACE BODY  
IN A DENSITY STRATIFIED OCEAN: USER'S GUIDE**

by  
M.A. Carroll  
EBOR Computing  
in collaboration with E.O. Tuck - University of Adelaide and  
the following DSTO staff: R. Webster, G. Furnell, A. Legg.

**Abstract**

This report provides details of the requirements needed to run the computer program WAKE. WAKE was developed during the period February-March 1992 as a result of a research project which was undertaken within DSTO-Salisbury. WAKE calculates the internal wave velocity field generated by a moving subsurface body in a density stratified ocean.

# **INTERNAL WAVES IN SUBMARINE WAKES**

M.A. Carroll

**EBOR COMPUTING**

**Defence Science & Technology Organisation  
Salisbury, S.A. 5108**

19 June 1992

**Table of Contents**

1. Introduction .....	3
2. Running Wake .....	4
2.1. Parameter File .....	4
2.2. Density Profile .....	7
3. Output .....	8
3.1. Velocity Component Output File .....	8
3.2. Integration File .....	13
4. MATLAB .....	14
4.2. 2-D Wave Plots .....	16
4.3. Matrix Intensity Plots .....	16
4.4. Contour and Field Flow Plots .....	17
4.4.1. Contour Plots .....	18
4.4.2. Field Flow Plots .....	18
4.5. Printing .....	19
5. Compiling .....	21
6. Program Description .....	23
6.1. Stage 1 : Initialisation .....	23
6.2. Stage 2 : Binding the roots of $D(\theta, k)$ .....	23
6.3. Stage 3 : Solving the ODE .....	27
6.3.1. Amplitude Functions $A(\theta)$ , $B(\theta)$ .....	28
6.4. Stage 4 : Integrands and Integration .....	29
7. Testing .....	31
8. Error Messages .....	37
9. Related Documents .....	40

### 1. Introduction

From February - March, 1992 Professor Ernie Tuck from the Applied Mathematics Department of Adelaide University undertook an investigation for DSTO of Submarine Wakes. The emphasis of the investigation was to compute the internal waves generated by the passage of the submarine through a stratified ocean. Ebor Computing was engaged to provide software support to Professor Ernie Tuck and DSTO. This support consisted of developing software to model these internal waves and produce output in a form so that it may be studied or used by graphics packages such as MATLAB and PLOTZ to display images of the wakes.

Figure 1 is a diagram showing the overall structure of the software developed.

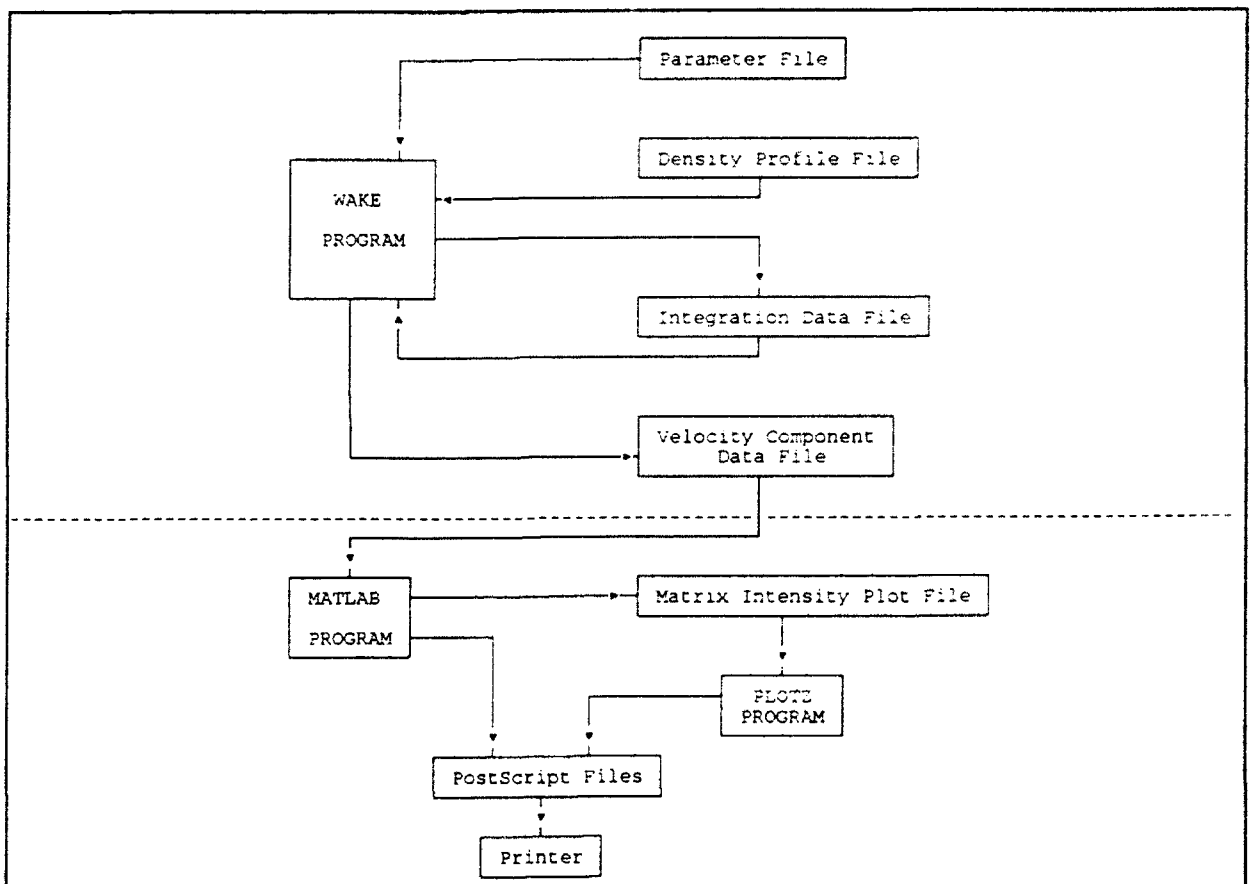


Figure 1 : System Diagram

This document describes the program developed by Professor Ernie Tuck, Dr. Graham Furnell, Mr. Tony Legg from DSTO and Mr. Michael Carroll of Ebor Computing.

## 2. Running Wake

The program was developed on a SparcStation using MATLAB and PLOTZ to generate plots of the wakes. To run the program you need to copy all the source, parameter and density profile files into your directory and compile them, see Chapter 5 on Compiling. You will also need access to the IMSL Library Routines (Fortran Version). Once the program has been compiled the program is run as follows :

> wake

### 2.1. Parameter File

The user is not prompted for any inputs, all inputs to the program are contained in the parameter file WAKE.PAR. Figure 2 shows a sample parameter file :

**NB :** The Parameter file must have the following format :

- 3 Header/Comment lines :** Gives information such as file purpose and date.
- 23 Parameter Lines :** First 71 Characters contain a comment on the parameter, the rest of line contains the actual parameter.

```

This file is read by WAKE and contains initial values for some of WAKE's
options.
Michael Carroll 23/4/92

NUMTHETA - The number of theta values to calculate data for (<=30) : 30
MODE      - The mode to calculate data for : 1
THETAMAX  - The maximum theta value allowed (<= Pi/2) : 1.56
SUBDTH    - The depth of the submarine (in metres) : 50.
SUBSPD    - The speed of the submarine (in metres/second) : 2.5
SUBLEN    - The length of the submarine (in metres) : 100.
SUBRAD    - The radius of the submarine (in metres) : 5.
XSTART    - The starting point for the X-Grid (in metres) : 1000.
XEND      - The end point for the X-Grid (in metres) : 1000.
XSTEP     - The step size in X (in metres) : 0.
YSTART    - The starting point for the Y-Grid (in metres) : 0.
YEND      - The end point for the Y-Grid (in metres) : 2000.
YSTEP     - The step size in Y (in metres) : 10.
ZSTART    - The starting point for the Z-Grid (in metres) : 0.
ZEND      - The end point for the Z-Grid (in metres) : 480.
ZSTEP     - The step size in Z (in metres) : 10.
DEPTH     - The depth of the ocean : 480.
DENFIL    - Name of the file containing the density profile info. : dawson.pro
NUMDEN    - Number of points in the density profile, DENFIL : 97
DBGFIL    - Name of the file to write the debugging information to : debug.dat
VELFIL    - Name of file to receive output velocity component data : arrow.dat
INTFIL    - Name of the file to receive integration data : int.dat
COMPU     - Compute the U component velocity ?? (Y) or (N) : N
COMPV     - Compute the V component velocity ?? (Y) or (N) : Y
COMPW     - Compute the W component velocity ?? (Y) or (N) : Y

```

**Figure 2 : Sample Parameter File**



The parameters read in convey the following information:

- NUMTHETA :** An integer in the range 2..30. NUMTHETA describes the number of theta values to divide the interval  $\theta_{\text{min}} \dots \pi/2$  into. The amplitude functions are then calculated at those theta values.
- MODE :** An integer in the range 1..10. MODE specifies the internal wave mode that we wish to calculate the data for.
- MAXTHETA :** The maximum theta value allowed.  $\text{MAXTHETA} < \pi/2$ .
- SUBDTH :** A real in the range 0..Depth of the Ocean. SUBDTH specifies the depth of the submarine in metres. The Depth of the Ocean is the input parameter DEPTH.
- SUBSPD :** A real in the range 0..Maximum Speed. SUBSPD specifies the speed of the submarine in metres/second. Maximum speed is set at 30 m/s.
- SUBLEN :** A real. SUBSPD specifies the length of the submarine in metres.
- SUBRAD :** A real. SUBRAD specifies the radius of the submarine in metres.
- XSTART :** A real. XSTART specifies the point in the X-Plane at which the integrations will be started from.
- XEND :** A real. XEND specifies the point in the X-Plane at which the integrations will end.  $\text{XEND} > \text{XSTART} > 0$
- XSTEP :** A real. XSTEP specifies the size of the X-Grid (i.e. the spacing between consecutive points in the X-Plane at which an integration will be calculated.  $\text{XSTEP} > 0$
- YSTART :** A real. YSTART specifies the point in the Y-Plane at which the integrations will be started from.
- YEND :** A real. YEND specifies the point in the Y-Plane at which the integrations will end.  $\text{YEND} > \text{YSTART} > 0$
- YSTEP :** A real. YSTEP specifies the size of the Y-Grid (i.e. the spacing between consecutive points in the Y-Plane at which an integration will be calculated.  $\text{YSTEP} > 0$

- ZSTART :** A real. ZSTART specifies the point in the Z-Plane at which the integrations will be started from. (ie first depth position)  
ZSTART must be a multiple of :  $DEPTH / ((NUMDEN-1)/2)$
- ZEND :** A real. ZEND specifies the point in the Z-Plane at which the integrations will end (ie last depth position).  
ZEND must be a multiple of :  $DEPTH / ((NUMDEN-1)/2)$
- ZSTEP :** A real. ZSTEP specifies the size of the Z-Grid (i.e. the spacing between consecutive points in the Z-Plane at which an integration will be calculated.  
ZSTEP must be a multiple of :  $DEPTH / ((NUMDEN-1)/2)$
- DEPTH :** The depth of the ocean as described by the density profile.  $DEPTH > 0$
- DENFIL :** A character string containing the name of the file which holds the density profile.
- \*\*\*\* DENSITY PROFILE MUST HAVE AN ODD \*\*\*\*  
\*\*\*\* NUMBER OF POINTS \*\*\*\*
- NUMDEN :** The number of points in the density profile, DENFIL.
- DBGFIL :** A character string containing the name of the file to receive the debugging information.
- VELFIL :** A character string containing the name of the file to receive the velocity component data.
- INTFIL :** A character string containing the name of the file to receive the integration data.
- COMPU :** A character indicating whether the U component velocity should be calculated. "Y" or "y" indicates "YES!, Calculate the U component" "N" or "n" indicates "No!, Calculation of the U component is not required".
- COMPV :** A character indicating whether the V component velocity should be calculated. "Y" or "y" indicates "YES!, Calculate the V component" "N" or "n" indicates "No!, Calculation of the V component is not required".
- COMPW :** A character indicating whether the W component velocity should be calculated. "Y" or "y" indicates "YES!, Calculate the W component" "N" or "n" indicates "No!, Calculation of the W component is not required".

**NB :** The number of points in the density profile, NUMDEN, is linked implicitly with the depth of the ocean, DEPTH. (NUMDEN-1)/2 should divide DEPTH evenly  
i.e. ( DEPTH modulo (NUMDEN-1)/2 ) = 0

### 3. Output

The program produces an intermediate output file containing the integration data, but the main output file contains the velocity component data.

#### 3.1. Velocity Component Output File

The velocity component output file contains a stream of six (6) columns of numbers which can be used by MATLAB (or other graphing utilities) to create several different type of graphs. Figure 4 shows a sample velocity component output file :

100.	0.	0.	0.	0.00000E-00	-6.92253E-07
100.	10.	0.	0.	7.92248E-04	-5.79579E-07
100.	20.	0.	0.	1.17258E-03	-3.12313E-07
100.	30.	0.	0.	1.11829E-03	-4.18162E-08
100.	40.	0.	0.	8.22264E-04	1.45645E-07
100.	50.	0.	0.	4.73894E-04	2.32705E-07
100.	60.	0.	0.	1.83084E-04	2.40433E-07
100.	70.	0.	0.	-1.63455E-05	2.07768E-07
100.	80.	0.	0.	-1.41567E-04	1.55930E-07
100.	90.	0.	0.	-2.05422E-04	7.54386E-08
100.	100.	0.	0.	-2.11459E-04	8.57241E-08
200.	0.	0.	0.	0.00000E-00	-6.96273E-07
200.	10.	0.	0.	8.92248E-04	-3.76574E-07
200.	20.	0.	0.	1.47278E-03	-2.14316E-07
200.	30.	0.	0.	1.14799E-03	-4.58662E-08
200.	40.	0.	0.	8.52764E-04	2.46645E-07
200.	50.	0.	0.	4.70934E-04	4.63655E-07
....	...	..	..	.....	.....
....	...	..	..	.....	.....
....	...	..	..	.....	.....
1000.	60.	480.	0.	1.41223E-06	-6.67499E-07
1000.	70.	480.	0.	1.42418E-06	-8.10197E-07

Figure 4 : Sample Velocity Component Output File

It consists of 6 columns of data. The last three columns correspond to the integrals of U,V and W, respectively, at the point X,Y,Z specified in the first three columns. (NB: in this particular run of WAKE only the V and W velocity components were calculated, the U column therefore contains only zeroes). It is up to MATLAB or any other graphing utility to separate the data correctly and produce meaningful graphs. See Section 4 : MATLAB.

Several different types of graphs can be produced. Figure 5 shows a 2-D Wave Plot and a 3-D mesh plot of the V Velocity Component taken at the ocean surface with the X-Y grid extending 6 and 2 kilometres respectively. Figure 6 shows a Matrix Intensity Plot, produced using PLOTZ, XV and MATLAB. Figure 7 shows the V and W Component Velocity Contour Plot. Figure 8 shows a zoomed in image of Figure 7 with arrows indicating the field flow.

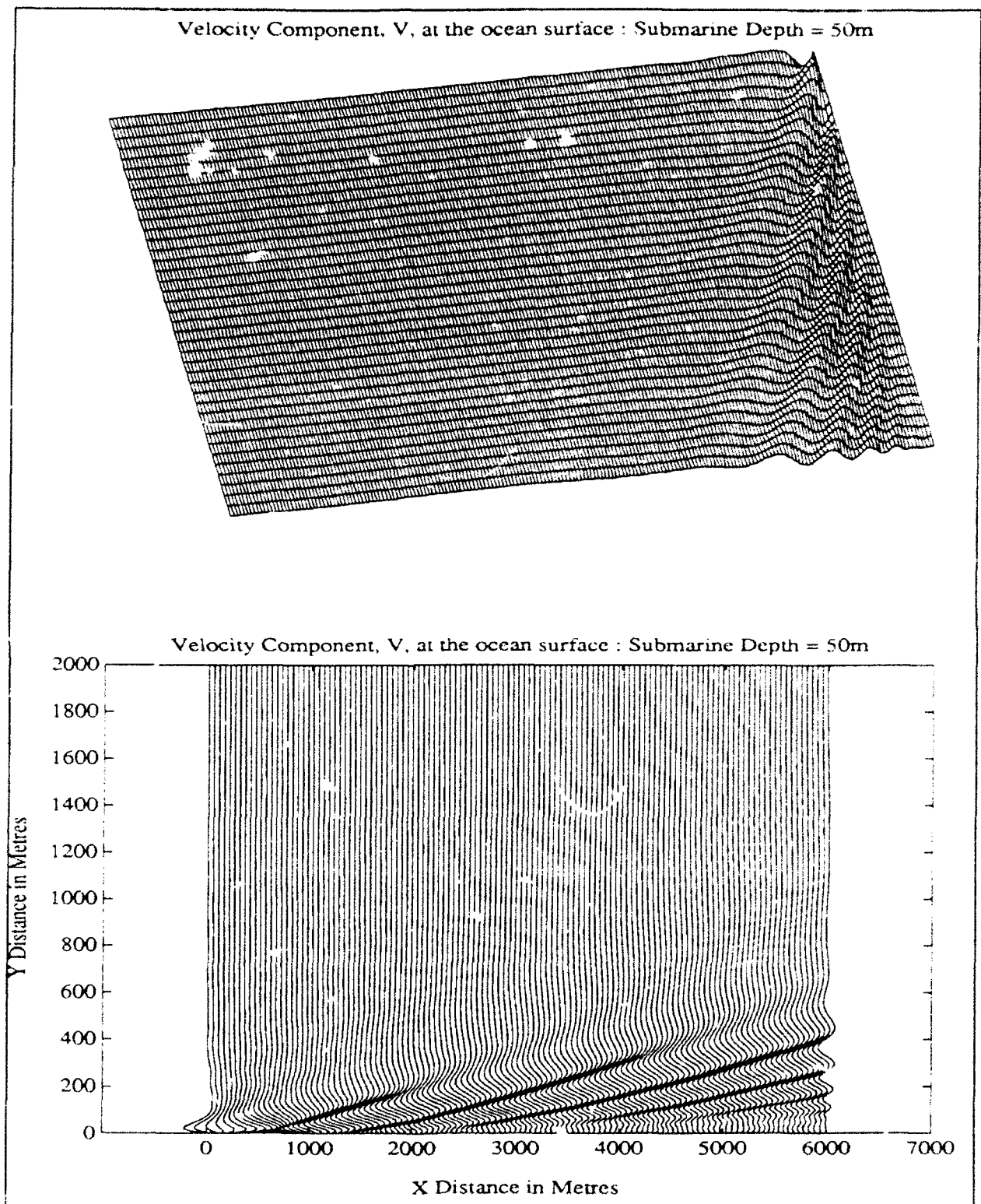


Figure 5 : 3-D Mesh and 2-D Wave Plots

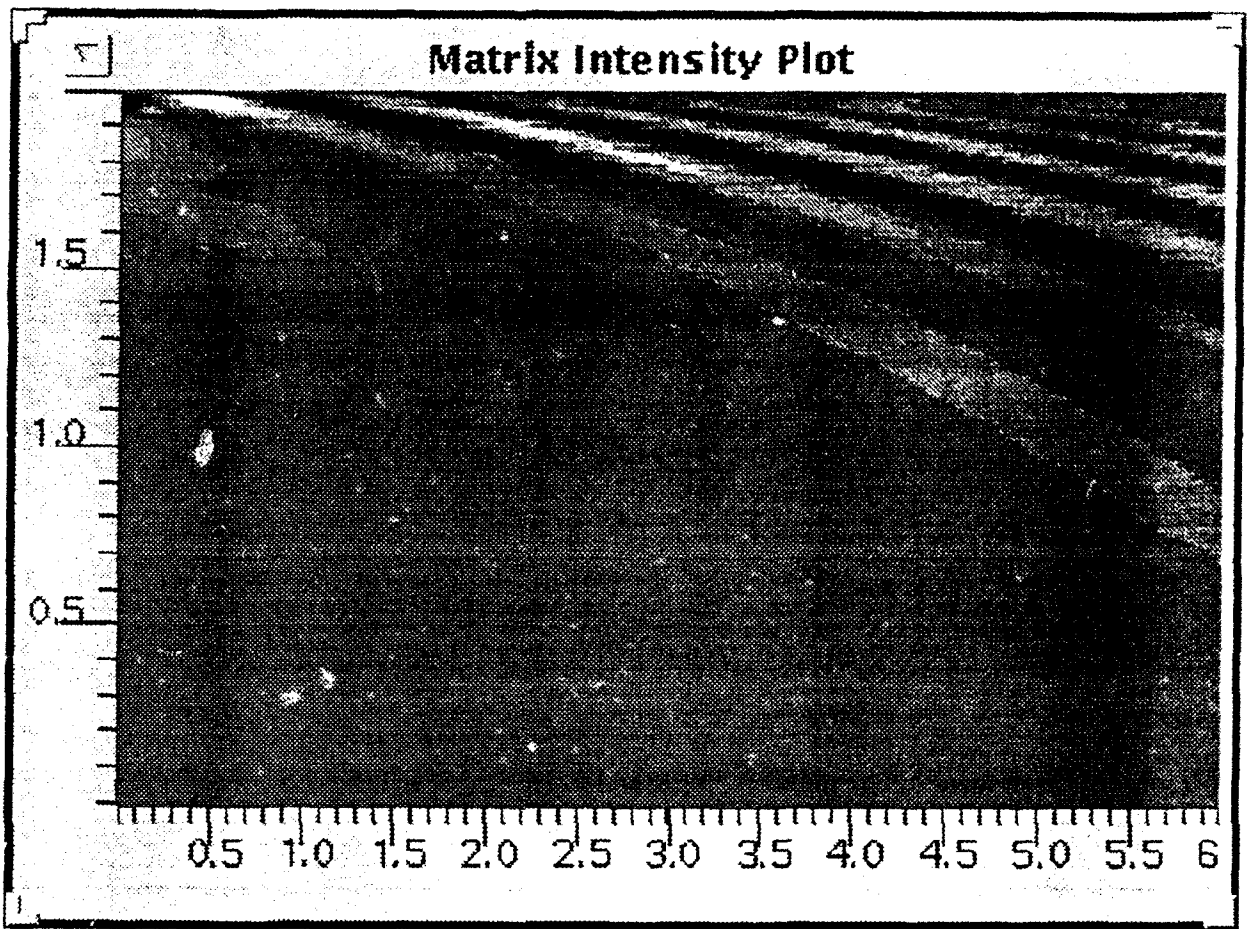


Figure 6 : Matrix Intensity Plot

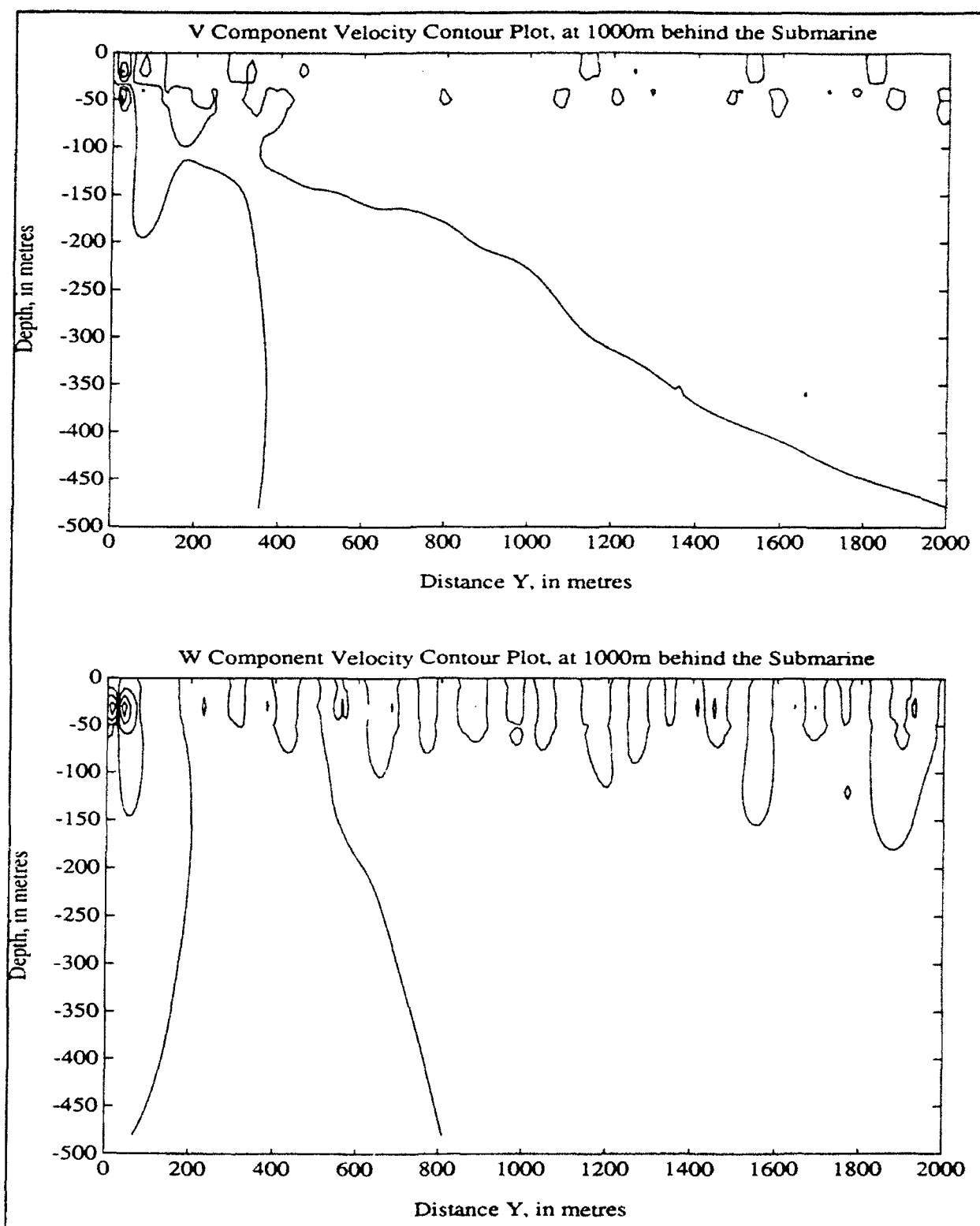


Figure 7 : V and W Contour Plots

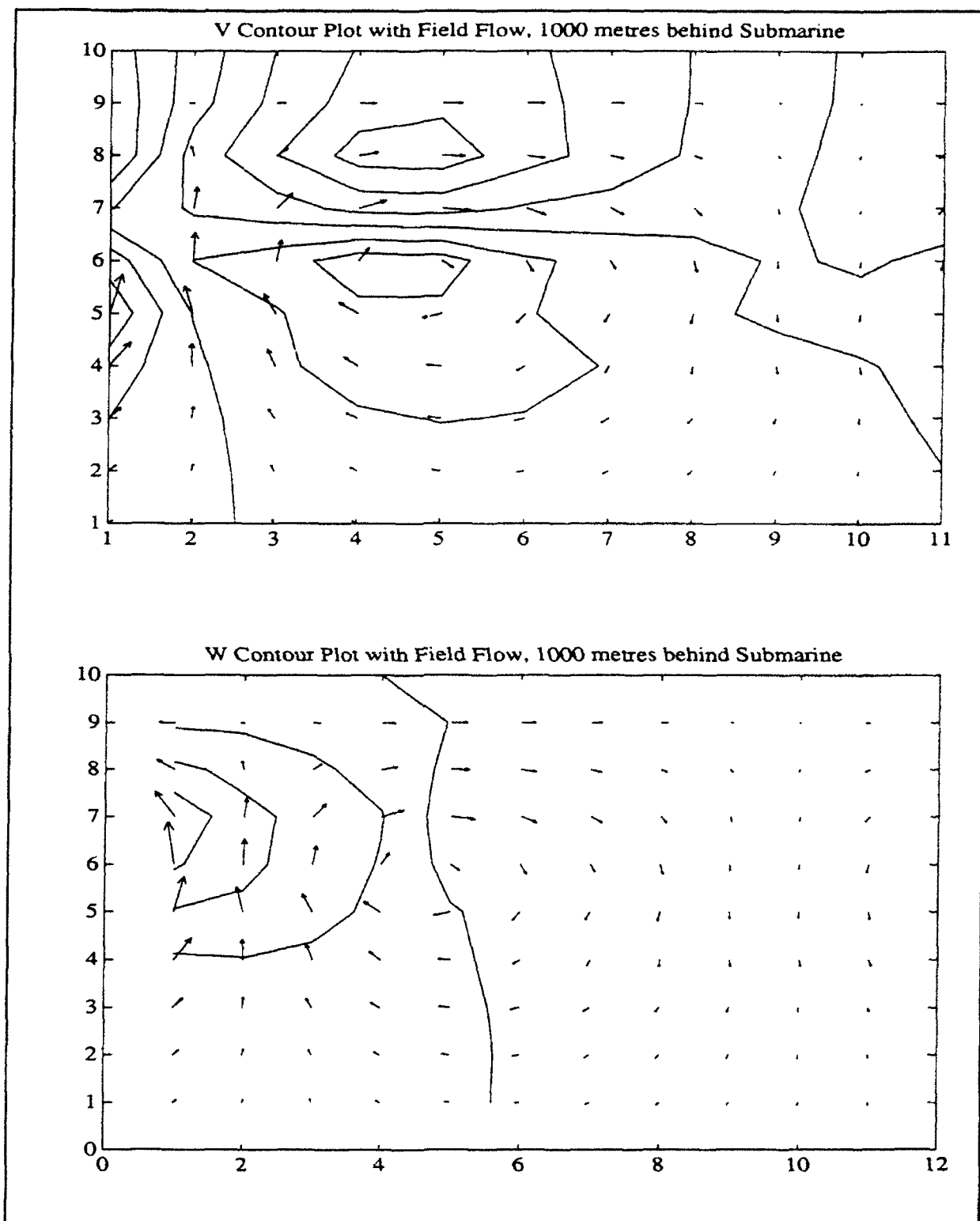


Figure 8 : Zoomed Image of Contour Plot with Field Flow Indicators



### 3.2. Integration File

The intermediate output produced by WAKE is the integration file. This file contains the data required for the integration. It consists of two parts, a Header and a Main Body.

**Header :** The header contains some of the parameters used in calculating the amplitude functions. These are :

Mode Number, Number of Depth Values, Depth Step,  
Number of  $\theta$  Values, Submarine Depth and Speed.

**Main Body :** The main body contains for each  $\theta, k$  pair the amplitude functions  $A(\theta)$  and  $B(\theta)$  for all depth values.

Figure 9 shows a sample integration data file :

Mode Number 1 Number of Z Values 49 Z Step 10.00000000000000 Number of Theta Values 30 Theta Step 1.3501382294919D-02 Submarine Speed 2.500000000000000 Submarine Depth 50.00000000000000				HEADER  Contains the parameters  used to generate the  Amplitude Functions
Theta 0.955754:5794732 Wave Number, K 4.9903212012139D-18 A(Theta) -2.5419091178105D-20 -2.5413779291835D-20 -2.5397766197426D-20 -2.5370978888899D-20 ..... -3.2228172003193D-21 -1.5948747568649D-21 3.4597398653662D-23 ..... ..... .....	B(Theta) -2.5419091178105D-20 2.1288060573866D-07 4.2966042338244D-07 6.4307667182042D-07 ..... 3.2497013160336D-05 3.2650882233180D-05 3.2652830218816D-05 ..... ..... .....	Z -480.000000000000 -470.000000000000 -460.000000000000 -450.000000000000 ..... -20.000000000000 -10.000000000000 0.000000000000 ..... ..... .....	MAIN BODY  Repeated NUMTHETA times  from $\theta_{min} \dots \pi/2 - \theta_{step}$	
Theta 1.5502949445000 Wave Number, K 0.65904859979995 A(Theta) 1.9842958911445-101 3.0538448560643D-99 6.0492250256296D-97 ..... -2.7622322606383D-07 -2.5742411595178D-09 -1.3120418273929D-11 4.2717012874524D-17	B(Theta) 1.9842958911445-101 5.5573161700118D-99 1.1051914847512D-96 ..... 2.4345912765042D-07 4.0750962927742D-09 2.4372036970908D-11 2.4183772601375D-13	Z -480.000000000000 -470.000000000000 -460.000000000000 ..... -30.000000000000 -20.000000000000 -10.000000000000 0.000000000000		

Figure 9 : Sample Integration Data File

4. MATLAB

To generate the graphs shown in Figures 5,6,7,8 requires a graphics package. The one described here is called MATLAB and can be run both on PC's and on SUN WorkStations.

To get started, invoke the matlab package with the command :

**> matlab**

After a moment, the MATLAB banner and a prompt like ">>" will appear. The MATLAB interpreter is awaiting instructions. Load in the velocity component data file generated by WAKE as follows :

**>> load <filename>**

After the file has been loaded, find out the size of the matrix that MATLAB has loaded the data into by executing the command :

**>> whos**

MATLAB will return with an answer that looks something like :

```
dolphin> matlab

      < P R O - M A T L A B >
(c) Copyright The MathWorks, Inc. 1984-1991
      All Rights Reserved
      Version 3.5i      18-Jul-1991

      HELP, DEMO, INFO, and TERMINAL are available
>>
>> load output.dat
>> whos
              Name          Size          Total          Complex
              output        5015 by 6      30090             No

Grand total is (30090 * 8) = 240720 bytes,
>>
```

**Figure 10 : Finding out about current variables and their sizes**

Notice the matrix is a six column matrix. To generate diagrams like Figure 5 and Figure 6 requires reshaping the matrix into an appropriate format. This is done as follows :

```
>> u = reshape(filename(:,4),m,n) ;  
>> v = reshape(filename(:,5),m,n) ;  
>> w = reshape(filename(:,6),m,n) ;
```

where :

FILENAME = Name of the data file generated by WAKE without its file extension  
The ":" symbol means "all". Thus, A(:,4) means extract all rows of the fourth column of the matrix A, whereas A(4,:) means extract all columns of the fourth row of the matrix A.

M = Number of Y-Grid points generated by WAKE  
(i.e.  $M = ((YEND - YSTART) / YSTEP) + 1$ )

N = A number. N is the number of X-Grid points generated by WAKE  
(i.e.  $N = ((XEND - XSTART) / XSTEP) + 1$ )

U = U is the resultant M x N matrix corresponding to the U component velocity data (if COMPU = 'Y' ie WAKE has generated the data for the U velocity component)

V = V is the resultant M x N matrix corresponding to the V component velocity data (if COMPV = 'Y' ie WAKE has generated the data for the V velocity component)

W = W is the resultant M x N matrix corresponding to the W component velocity data (if COMPW = 'Y' ie WAKE has generated the data for the W velocity component)

#### 4.1. 3-D Mesh Plots

To generate a 3-D Mesh Plot of the V Velocity Component as shown in Figure 5, execute the following command :

```
>> mesh(v,[hr,e])
```

where :

V = MxN matrix containing the V velocity component data  
HR = Horizontal Rotation  
E = Elevation

NB : A viewpoint matrix [HR,E] of [80,80] is recommended as a start.

A title may be put on the plot as follows :

```
>> title('3-D Mesh Plot')
```

4.2. 2-D Wave Plots

To generate a 2-D Wave Plot of the V Velocity Component as shown in Figure 5, execute the following commands :

First generate an offset matrix as follows :

```
>> offset = ones(M,1)*linspace(0,TOP,N)/100000. ;
```

where

N	= Number of columns in our matrix V
M	= Number of rows in our matrix V
TOP	= Arbitrary scale for the Y-Axis of the plot.

Next generate a scaling matrix as follows :

```
>> scale = linspace(ymin,ymax,M) ;
```

where

N	= Number of columns in our matrix V
YMIN	= Starting value of the Y-Axis
YMAX	= End value of the Y-Axis

Now define a new matrix, X which is the matrix V with each column offset slightly as follows :

```
>> x = v + offset ;
```

The 2-D Wave Plot, with an appropriate title, is generated in the Graphics Window with the following commands :

```
>> plot(x,scale,'b-')  
>> title('2-D Wave Plot')
```

4.3. Matrix Intensity Plots

To generate a Matrix Intensity Plot of the V Velocity Component like Figure 6, requires a package called PLOTZ. First save the matrix A to a file. This is done by executing the command

```
>> save wave v
```

This generates a file called WAVE.MAT, in the current directory, which contains the data from the matrix A. Now exit from MATLAB with the command

```
>> quit
```

To start up PLOTZ and generate the matrix intensity plot execute the following command:

```
> plotz wave.mat [1,2,3,4] xmin xmax ymin ymax
```

where

WAVE.MAT = File generated by MATLAB containing the velocity component data  
 [1,2,3,4] = Choose one of the following options :  
           1 = Grey  
           2 = Colour  
           3 = 4 Tones of Green  
           4 = Black & White (default)  
 XMIN     = Starting value of the X-Axis  
 XMAX     = End value of the X-Axis  
 YMIN     = Starting value of the Y-AXIS  
 YMAX     = End value of the Y-AXIS

#### 4.4. Contour and Field Flow Plots

To generate the contour and quiver plots as shown in Figure 7 and Figure 8, run WAKE fixing X at some point behind the submarine, generating the V and W component velocities in the Y-Z Plane. First load the data and reshape the matrix into an appropriate format as follows :

```
>> load <filename>

>> v = reshape(filename,5),m,n) ;
>> w = reshape(filename(:,6),m,n) ;
```

where :

FILENAME = Name of the data file generated by WAKE without its file extension  
 M        = Number of Y-Grid points generated by WAKE  
           (i.e.  $M = ((YEND - YSTART) / YSTEP) + 1$ )  
 N        = A number, N is the number of Z-Grid points generated by WAKE  
           (i.e.  $N = ((ZEND - ZSTART) / ZSTEP) + 1$ )  
 V        = V is the resultant M x N matrix corresponding to the V component  
           velocity data (if COMPV = 'Y' ie WAKE has generated the data for  
           the V velocity component)  
 W        = W is the resultant M x N matrix corresponding to the W component  
           velocity data (if COMPW = 'Y' ie WAKE has generated the data for  
           the W velocity component)

4.4.1. Contour Plots

Set the scale on the Y and Z axis to correspond to the data generated.

```
>> yscale = linspace(ystart,yend,M) ;  
>> zscale = linspace(-zend,zstart,N);
```

Plot and keep the empty axis as follows :

```
>> plot([ystart yend], [-zend zstart],'.')  
>> hold on
```

Plot the V component velocity contour as follows :

```
>> vt = v';  
>> contour(vt,yscale,zscale,'b-')
```

VT is now the transpose of the matrix v. This is required to get the correct orientation. Appropriate titles and axis labels are added with the following commands :

```
>> title('V Component Velocity Contour Plot, at 1000m behind the Submarine')  
>> xlabel ('Distance Y, in metres')  
>> ylabel ('Depth, in metres')
```

4.4.2. Field Flow Plots

To produce a contour plot with field flow arrows overlayed, display in the Graphics Window the contour plot and execute the following commands :

```
>> hold on  
>> quiver(vt,wt)
```

where VT and WT are the transposes of the V and W velocity component matrices. This will produce a contour plot with field flow arrows overlayed. The arrows are scaled so that they indicate not only direction but magnitude. The scaling of the arrows become too small if we are looking at a wide area, hence it is necessary to look at small portions of the plot at any one time to see the structure of the field flow.

To look at a section of the plot like Figure 8 execute the following :

```
>> clg
>> hold off
>> contour(vt(zindstart:zindend,yindstart:yindend),'b-')
>> hold
>> quiver(vt(zindstart:zindend,yindstart:yindend),
          wt(zindstart:zindend,yindstart:yindend))
>> title('V Contour Plot with Field Flow')
```

Where :

ZINDSTART,ZINDEND,YINDSTART,YINDEND define the submatrix of the matrix VT.

This has the effect of zooming in on a portion of the plot so that we can see the structure of the field flow.

#### 4.5. Printing

To print within MATLAB, display the graph required in the graphics window and then execute the following commands :

```
>> meta <filename>
```

This creates a file called FILENAME.MET in the current directory. Create a postscript file of the plot as follows :

```
>> !gpp filename.met -dps -ol
```

This generates a postscript file of the plot which can be sent to a postscript printer in the usual manner.

For more information on MATLAB and MATLAB commands consult the MATLAB user manual.

To print a matrix intensity plot like Figure 6, save the plot generated by PLOTZ, as follows :

- 1). Click on the plot with the right button of the mouse
- 2). Select **Save to Raster**
- 3). Quit out of PLOTZ by clicking again on the plot with the right mouse button and selecting **Quit**

PLOTZ has now generated a Raster file called "PLOTZ.RAS". To print this file convert it to PostScript format as follows :

- 1). Execute the Raster file viewing package XV as follows :

*> xv plotz.ras*

- 2). Click with the right button of the mouse on the plot. This brings up a menu with various options and parameters.
- 3). Select **Save**. This brings up another menu with more options.
- 4). Select **PostScript** and rename the output file (if desired) and click on **OK**
- 5). This brings up a final menu with all the PostScript options (i.e. portrait, landscape, paper size, etc). Select the parameter desired and click on **OK**
- 6). A rotating fish indicates that XV is converting the Raster file to a Postscript file.
- 7). Quit out of XV and send the PostScript file just generated to a PostScript printer in the usual manner.



5. Compiling

To compile the WAKE program the following files need to be copied into a working directory. Access to the IMSL library Routines DQDAG, CSAKM, CSVAL (fortran version) is also required.

<b>MAKEFILE :</b>	Compiles all the routines into a library and links them all together. Figure 12 shows a sample makefile.
<b>COMP :</b>	A script file which compiles the source fortran files to object files and creates/updates a library called WAKE which contains all the necessary routines and functions. Figure 11 shows a sample compile file.
<b>DENSITY.DAT</b>	Name of the file which contains the density profile.
<b>WAKE.PAR</b>	Contains all of the parameters WAKE requires
<b>WAKE.F</b>	Main Program controls all the following subroutines
<b>BOUND.F</b>	Searches for changes of sign in the function $D(\theta, k)$ for a given $\theta$ and saves the intervals in a matrix
<b>COMPUTE_A.F</b>	Computes the Amplitude Functions $A(\theta)$ , $B(\theta)$ .
<b>COMPUTE_H.F</b>	Converts a real depth value into an integer grid point.
<b>DENSITY.F</b>	Reads in the density profile and computes $MU(z)$ .
<b>DET.F</b>	Returns the value of $D(\sigma, k)$ , where $D$ = Determinant of the Wronskian of the ODE.
<b>EXPANALYT.F</b>	Calculates the analytical solutions for an exponential density profile.
<b>FINDINTERVAL.F</b>	Finds an interval within which a root of the function passed occurs.
<b>FIND_ZEROES.F</b>	Returns the $\sigma(0)$ values.
<b>GETSIGVALS.F</b>	Uses a quadratic mapping technique to obtain an array of $\sigma$ values biased at the endpoints.
<b>INIT.F</b>	Reads in the initial parameters from the parameter file.
<b>INTEGRATE.F</b>	Performs the integrations using the IMSL routine DQDAG.
<b>LINEAR.F</b>	Performs a linear extrapolation to obtain a guess for the value of $K$ at the current $\sigma$ value using two previous $(\sigma, k)$ pairs.
<b>LOOKUP.F</b>	Looks up the interval matrix to obtain the intervals where roots of the function $D(\theta, k)$ occur.
<b>READINT.F</b>	Reads in the integration data for a fixed depth.
<b>RTBIS.F</b>	Numerical Recipes Bisection Method.
<b>RTSEC.F</b>	Numerical Recipes Secant Method.
<b>UINT.F</b>	Computes the $U$ velocity integrand.
<b>VINT.F</b>	Computes the $V$ velocity integrand.
<b>WINT.F</b>	Computes the $W$ velocity integrand.
<b>WRITEHEAD.F</b>	Writes the header information to the integration file.
<b>WRITEINT.F</b>	Writes the integration data to the integration data file.
<b>WFUNCNS.F</b>	Computes the functions $W_1$ , $W_1'$ , $W_2$ , $W_2'$ .
<b>COMMON.BLK</b>	Contains all of WAKE's common variables.

```
# This file compiles a fortran source file and puts the object into the library
# libnr.a      mac 17/2/93
f77 -g -c $1.f -o $1.o
ar r lib$2.a $1.o
ranlib lib$2.a
```

Figure 11 : Sample Compilation File

```
wake: wake.o bound.o compute_a.o compute_h.o density.o det.o expansiylt.o findinterval.o findzeroes.o getsigvals.o init.o integrate.o
linear.o lookup.o readint.o rtbis.o rtsec.o uint.o vint.o wint.o writehead.o writeint.o wfuncs.o

f77 -g wake.o -L. -lwake /home/shark/share/local/libnmslib.a -o wake

bound.o:      bound.f
              comp det wake
              ↳ Put path to RMSL Routines (fortran version) here !!

compute_a.o:  compute_a.f
              comp compute_a wake

compute_h.o:  compute_h.f
              comp compute_h wake

density.o:    density.f
              comp density wake

det.o:        det.f
              comp det wake

expansiylt.o: expansiylt.f
              comp expansiylt wake

findinterval.o: find_interval.f
                comp find_interval.f

findzeroes.o:  find_zeroes.f
                comp find_zeroes wake

getsigvals.o:  getsigvals.f
                comp getsigvals wake

init.o:        init.f
                comp init wake

integrate.o:   integrate.f
                comp integrate wake

linear.o:      linear.f
                comp linear wake

lookup.o:      lookup.f
                comp lookup wake

readint.o:     readint.f
                comp readint wake

rtbis.o:       rtbis.f
                comp rtbis wake

rtsec.o:       rtsec.f
                comp rtsec wake

uint.o:        uint.f
                comp uint wake

vint.o:        vint.f
                comp vint wake

wint.o:        wint.f
                comp wint wake

writehead.o:   writehead.f
                comp writehead wake

writeint.o:    writeint.f
                comp writeint wake

wfuncs.o:      wfuncs.f
                comp wfuncs wake

wake.o:        wake.f
                f77 -g -c wake.f -o wake.o
```

Figure 12 : Sample MAKEFILE

## 6. Program Description

The program can be divided into four main stages :

- STAGE 1 : Initialisation
- STAGE 2 : Binding the roots of the function  $D(\sigma, k)$
- STAGE 3 : Solve the O.D.E. and Calculate the Amplitude Functions  $A(\theta)$ ,  $B(\theta)$
- STAGE 4 : Form the 3 direction (x,y,z) integrands U,V,W and integrate them

These stages are described in the following sections.

### 6.1. Stage 1 : Initialisation

Stage 1 sets up the initial conditions. Figure 13 describes diagrammatically the structure for Stage 1

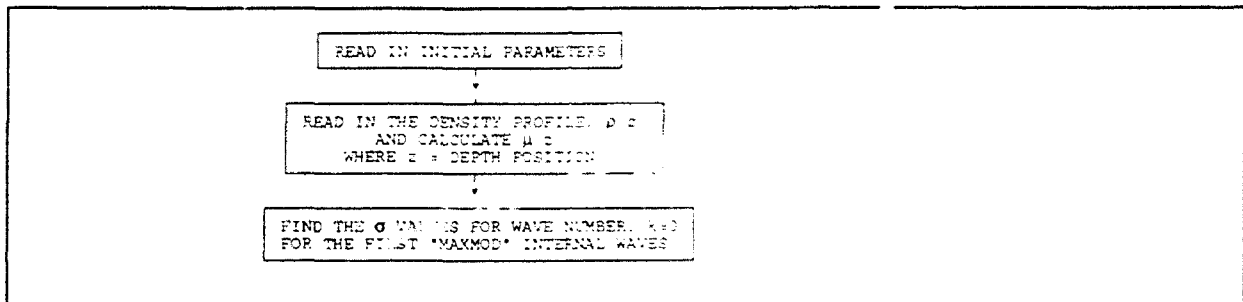


Figure 12 : Stage 1 :- Initialisation

The initial parameters are first read in from the parameter file, **WAKE.PAR**, see Section 2.1 for a description on the input parameter file. Next the program reads in from the file specified in the parameter, **DENFIL**, the density profile, see Section 2.2 for a description of the density profile.

### 6.2. Stage 2 : Binding the roots of $D(\theta, k)$

Once the initial parameters and density profile has been read in, consider the dispersion relation [see E.O. Tuck : Appendix 3 Submarine Internal Waves, 1992].

" Our concern here is very much with dispersion relations for internal waves, namely relations between wave speed  $c$  and wave number  $k$ . For convenience, we use instead of  $c$  a quantity proportional to its reciprocal square, namely

$$\sigma = \frac{g}{c^2}$$

where  $g$  is gravity. Then we need a connection between  $k$  and  $\sigma$ , e.g.  $k = K(\sigma)$ . One of our first tasks is to compute this relation for a given density distribution." [EOT 92]

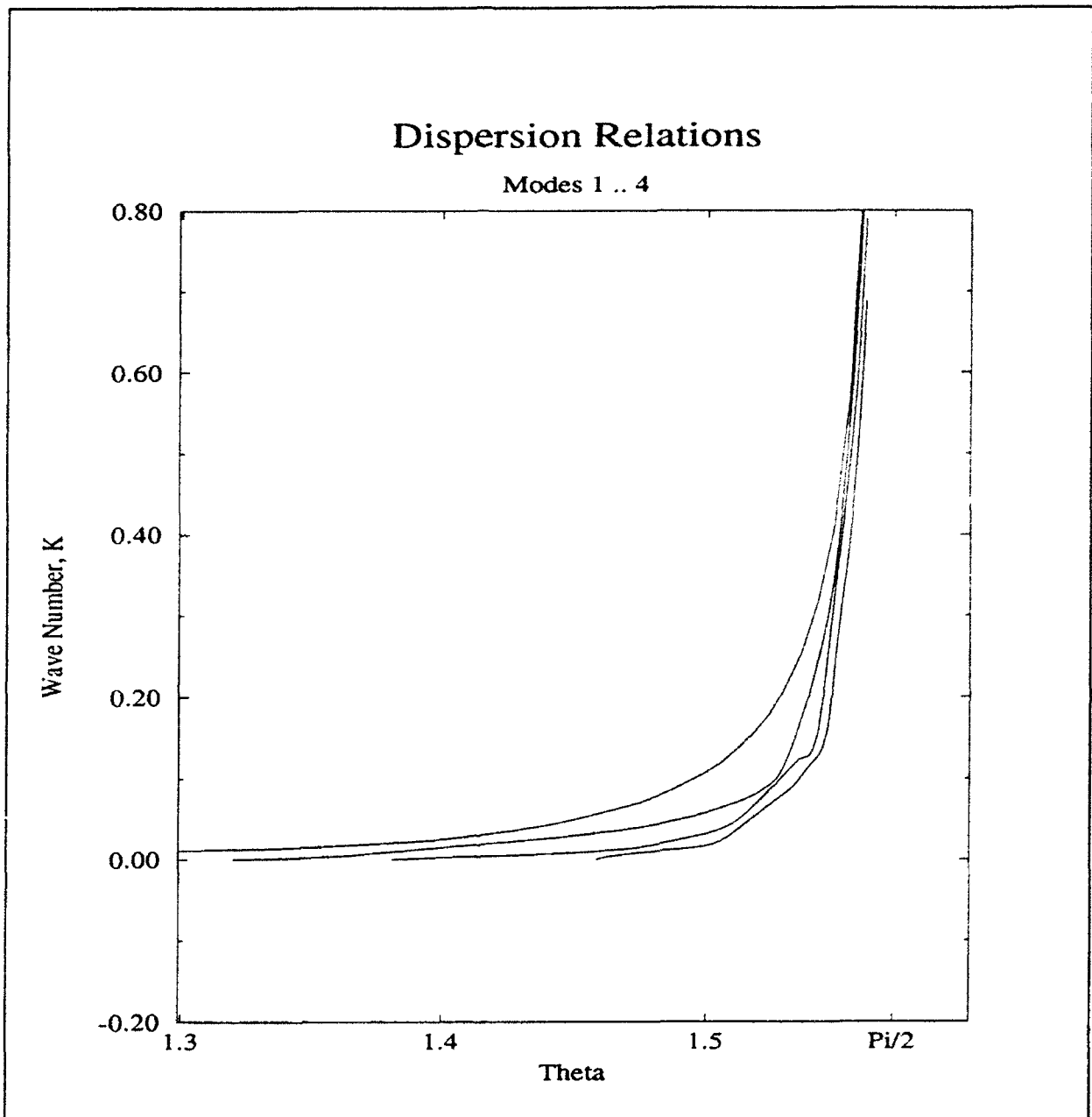


Figure 14 : Dispersion Relations

Figure 14 shows the dispersion relation for the first 4 internal waves (The unseen line  $\theta=k$  would correspond to the surface wave). We can see that as  $\theta \rightarrow \pi/2$  the graphs get very close together and at  $\theta = \pi/2$  there are in fact an infinite number of internal waves (this becomes a problem later when the program tries to pick out one particular internal wave and stay within that one mode).

Figure 15 shows diagrammatically the structure for Stage 2

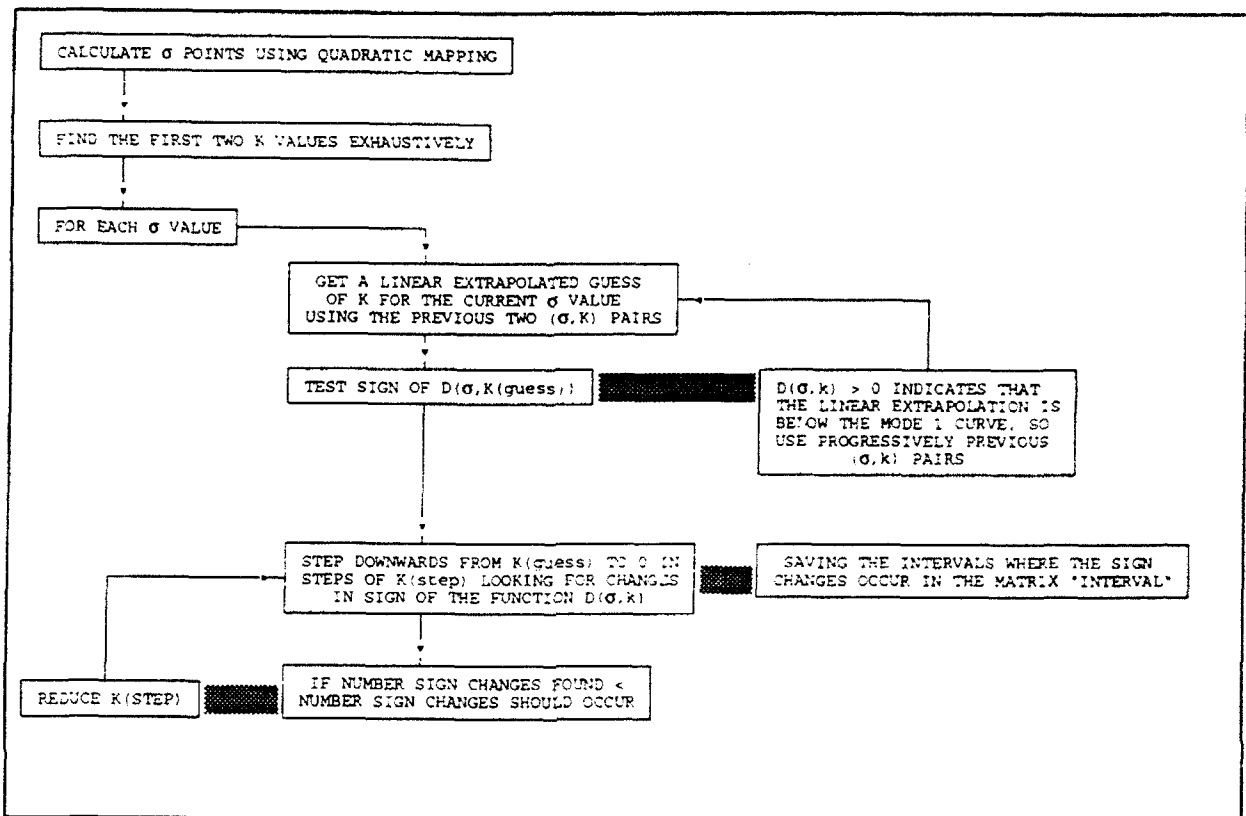


Figure 15 : Stage 4 : Binding the roots of  $D(\theta, k)$

The dispersion relation needs to be accurately determined for multiple modes. The first step is to accurately determine the first mode as this acts as an upper bound on the other modes. The first mode is determined in the following manner :

Each mode has a  $\theta_{\min}$ , denoted  $\theta^{\text{mode}}$  (e.g. the  $\theta_{\min}$  value for the first mode is denoted  $\theta^1$ ). Below this  $\theta_{\min}$  value internal waves do not exist for that mode (therefore below the  $\theta_{\min}$  value for the first mode, no internal waves exist). The interval  $\theta^1 \dots \text{THETAMAX}$  is divided into **NUMTHETA** subintervals, each of size  $\theta_{\text{step}}$ . The values of  $K$  for  $\theta^1$  and  $\theta^1 + \theta_{\text{step}}$  are determined by an exhaustive search.

Once these first two values have been determined a linear extrapolation guess,  $K_{\text{guess}}$ , for the value of  $K$  at the next  $\theta$  value can be obtained.

- [ It was found that perturbations in the dispersion curve resulted in the linear extrapolation projected below the first mode curve. To overcome this a test on the sign of the function  $D(\theta, k)$  at the point  $(\theta, K_{\text{guess}})$  is performed. If the function is negative then the extrapolation must be above the first mode, as desired (the probability of the extrapolation projecting into the third mode region where the function is also negative is considered to be negligible). If the function is positive then the extrapolation is below the first mode curve so the previous  $(\theta, k)$  pair is used for the extrapolation. This process of using progressively previous  $(\theta, k)$  pairs is repeated until a satisfactory linear extrapolated guess,  $K_{\text{guess}}$  has been obtained ]

The process is continued from  $\theta^1 + \theta_{\text{step}}$  through to **THETAMAX**, at each step using the two previous  $(\theta, k)$  pairs to obtain a linear extrapolated guess,  $K_{\text{guess}}$ , for the value of  $K$  at the next  $\theta$  value. Once  $K_{\text{guess}}$  has been obtained, the program searches downwards in steps of  $K_{\text{step}}$  looking for intervals where a sign change in the function  $D(\theta, k)$  occurs (i.e. searches for roots of the function  $D(\theta, k)$ ). For any given  $\theta$  value, the minimum number of sign changes that should occur is known through the determination of the  $\theta^{\text{mode}}$  values. If the number of sign changes found is less than the number of sign changes that should occur then the  $K_{\text{step}}$  value used must have been too large, so  $K_{\text{step}}$  is halved and the search for sign changes is repeated until the correct number of sign changes that should occur have been found. Once the correct number of sign changes have been found the intervals are saved in a matrix with the columns corresponding to the various  $\theta$  values and the rows containing the intervals where sign changes occur.

When the program wants to know the  $K$  value for a certain mode at a given  $\theta$  value all that is required is to look up in the **INTERVAL** matrix to get the interval where the sign change occurs for the given mode and use the Bisection Method to hone in on the root. If a higher mode than the first mode is selected then the program still calculates **NUMTHETA**  $\theta$  values between  $\theta^{\text{mode}}$  and **THETAMAX**, however, 10 preliminary calculation are performed from  $\theta^1$  to  $\theta^{\text{mode}}$ .

## 6.3. Stage 3 : Solving the ODE

Figure 16 shows diagrammatically the structure for stage 3

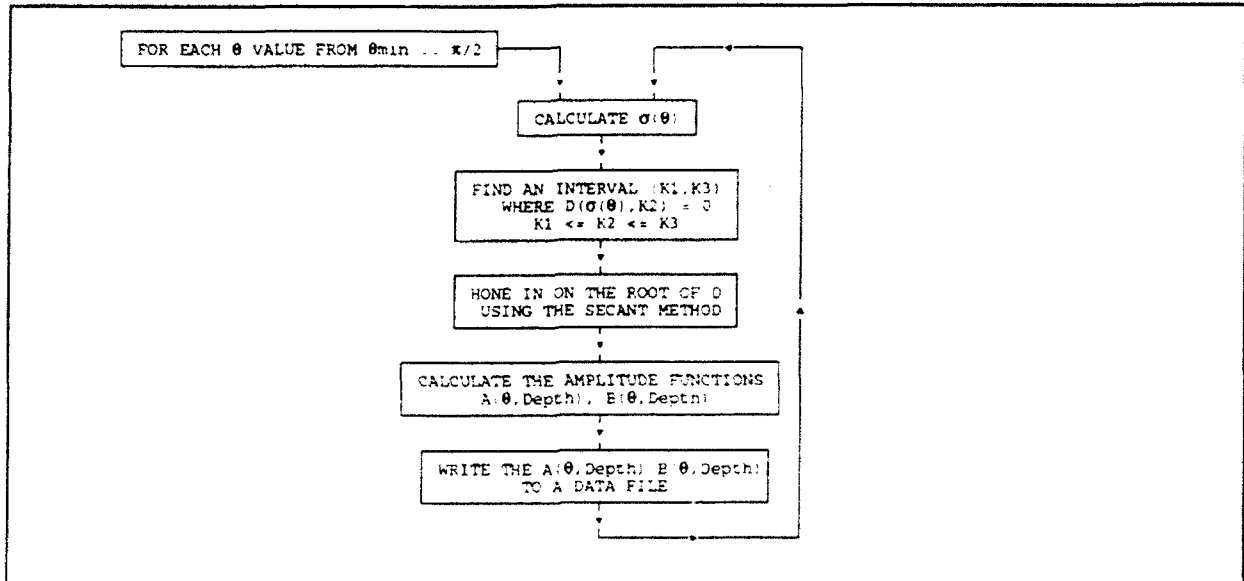


Figure 16 : Stage 3 :- Solving the ODE

In this stage we solve the ODE

$$(\rho W')' - (\kappa^2 \rho + \sigma \rho') W = 0$$

and calculate the amplitude functions  $A(\theta)$ ,  $B(\theta)$ .

First we divide the interval  $\theta_{\min} \dots \pi/2$  into NUMTHETA intervals and calculate  $\theta_{\text{step}}$ ,

$$\theta_{\text{step}} = \frac{\frac{\pi}{2} - \theta_{\min}}{\text{NUMTHETA}}$$

The Amplitude functions are then calculated at  $\theta = \theta_{\min} \dots (\pi/2 - \theta_{\text{step}})$  in steps of  $\theta_{\text{step}}$ .

So for each  $\theta$  we calculate the corresponding  $\sigma$  value,

$$\sigma = \kappa \sec^2(\theta)$$

By fixing  $\sigma$  and varying  $k$  we now find an interval  $k_1..k_3$ , where  $D(\sigma, k_2)=0$  with  $k_1 < k_2 < k_3$ . Once this interval has been found ( i.e. the root has been bounded) we use the Secant Method to hone in on the root. At this stage we have a  $\theta(\sigma)$  and corresponding  $k$  pair, we use this pair to calculate the Amplitude functions  $A(\theta)$ ,  $B(\theta)$  as shown in Section 6.3.1.

### 6.3.1. Amplitude Functions $A(\theta)$ , $B(\theta)$

For a detailed description of the method of calculating  $A(\theta)$  and  $B(\theta)$  see [E.O. Tuck 1992 : Submarine Internal Waves : Appendix 1]

NB: The amplitude functions are not defined explicitly in EOT 92, they appear implicitly in equation (4.1)

$$A(\theta) = \frac{k W_0}{\frac{\partial D(\theta, k)}{\partial k}} ; \quad B(\theta) = \frac{W_0'}{\frac{\partial D(\theta, k)}{\partial k}}$$

where

$$W_0(z) = \begin{cases} -W_2'(h) W_1(z) & : z \leq h \\ -W_1'(h) W_2(z) & : h \leq z \leq 0 \end{cases}$$

and

$h$  = Submarine Depth

$W = W_1(z)$  and  $W = W_2(z)$  are two separate solutions of the ODE

$$(\rho W')' - (k^2 \rho + \sigma \rho') W = 0$$

where

$$\sigma = \kappa \sec^2 \theta ; \quad \rho = \text{Density}$$

Once the Amplitude Functions have been calculated the data is written to the data file specified in the parameter INTFIL as specified in the parameter file "WAKE.PAR".



#### 6.4. Stage 4 : Integrands and Integration

In this stage of the program we form the directional velocity component integrands  $U$ ,  $V$ ,  $Z$  and integrate them. Figure 17 shows diagrammatically the structure for Stage 4

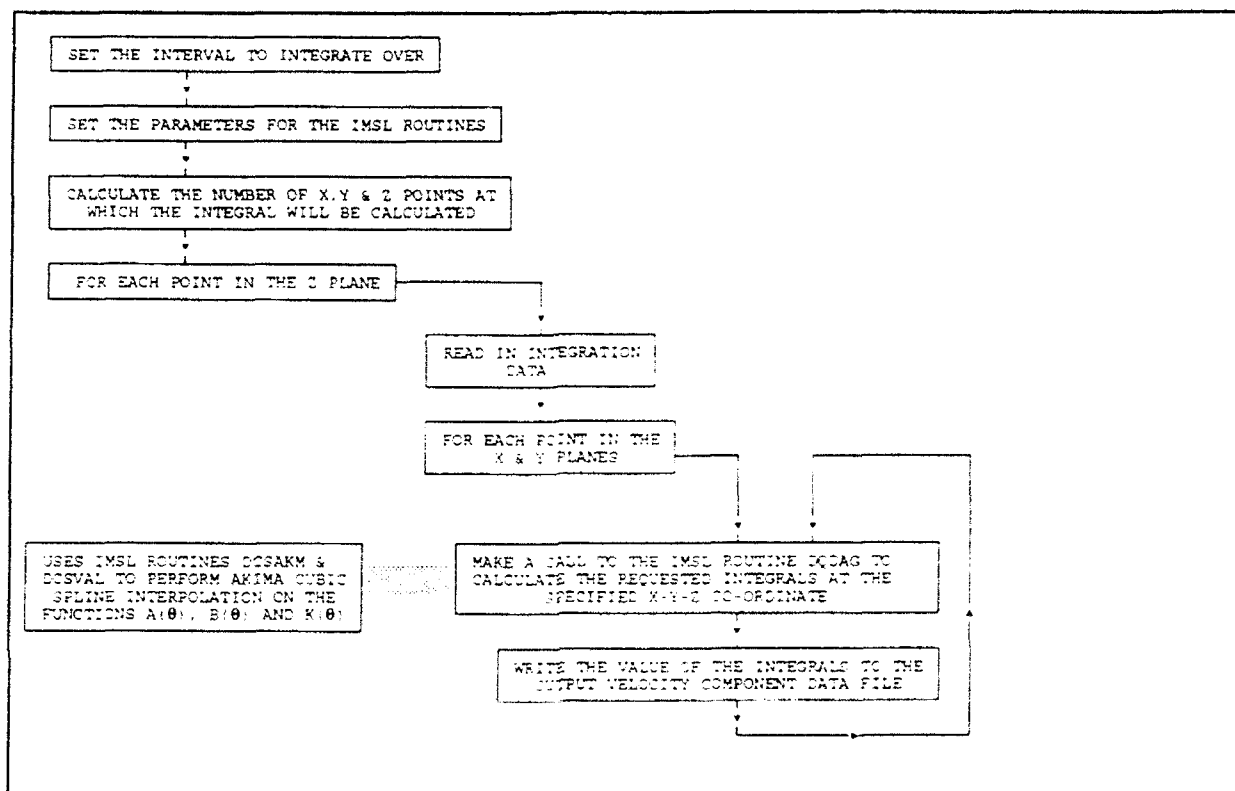


Figure 4 : Stage 4 :- Integrands and Integration

The integration is performed by the IMSL package routine **DQDAG**. This routine expects a continuous function (we achieve this by taking our discrete vectors which describe the functions  $A(\theta)$ ,  $B(\theta)$  and  $K(\theta)$  and performing Akima Cubic Spline Interpolations, using IMSL routines **DCSAKM** and **DCSVAL** as required to obtain continuous smooth functions) and an interval to integrate over (in our case  $\theta_{min} \dots \pi/2$ ). The IMSL integration routine then requires some desired accuracy and quadrature rules to be set. **DQDAG** is a general purpose integrator that uses a globally adaptive scheme in order to reduce the absolute error. It subdivides the integrating interval and uses a  $(2k+1)$ -point Gauss-Kronrod rule to estimate each subinterval. It was found that a Gauss-Kronrod Rule with 20 - 41 points, and a requested accuracy of  $1.D-7$  produced the best results.

The amplitude data  $A(\theta)$ ,  $B(\theta)$  is stored in the data file **INTFIL**. The amplitude data is read back into the program so that we have the arrays **A**, **B** containing the amplitude functions for all  $\theta$  values at a specific depth. (The file **INTFIL** contains the amplitude functions for all depths at a specific  $\theta$  value).

The first step is to define the Z-Grid over which the integrations will take place, then for each depth position (ie each point in the Z-Grid) the  $A(\theta)$ ,  $B(\theta)$  and  $K(\theta)$  is read in. The next step is to define the X-Y Grid over which the integrations will take place. For each point in the X-Y-Z subspace the integration is calculated for each of the requested components and the results are written to the output velocity data file as described by the parameter VELFIL.

The U velocity component corresponds to the X direction (ie in the same direction that the submarine is moving). The V velocity corresponds to Y direction (ie sideways to the direction that the submarine is moving). The W velocity corresponds to the Z direction (ie the depth direction). The following three equations describe the integrals of the three velocity components.

$$\frac{W}{U} = -\frac{2V}{\pi} \int_{\theta_{min}}^{\pi/2} \cos(kx \cos\theta) \cos(ky \sin\theta) \left[ \frac{\sin(k \cos\theta L/2)}{L/2} \right] A(\theta) d\theta$$

$$\frac{U}{U} = \frac{2V}{\pi} \int_{\theta_{min}}^{\pi/2} \sin(kx \cos\theta) \cos(ky \sin\theta) \left[ \frac{\sin(k \cos\theta L/2)}{L/2} \right] B(\theta) \cos\theta d\theta$$

$$\frac{V}{U} = \frac{2V}{\pi} \int_{\theta_{min}}^{\pi/2} \cos(kx \cos\theta) \sin(ky \sin\theta) \left[ \frac{\sin(k \cos\theta L/2)}{L/2} \right] B(\theta) \sin\theta d\theta$$

- where U = Submarine Speed
- V = Submarine Volume
- L = Submarine Length
- X = Position in the X-Plane relative to the submarine
- Y = Position in the Y-Plane relative to the submarine
- $A(\theta)$  = Amplitude Function, A
- $B(\theta)$  = Amplitude Function, B
- k = Wave Number

7. Testing

Stage 1 of the program consists of reading in parameters and the density profile. This section can be tested by using the DBXTOOL debugging utility and stepping through each line of the code checking that the correct values are read in to the right variables.

Stage 2 of the program is concerned with finding the roots of the function  $D(\theta, k)$ . The method finally chosen (i.e. Linear extrapolated guess and then downwards search for changes in sign) was only the last of many techniques investigated, (others included searches using the secant method, fixing either  $K$  or  $\theta$ ). This final technique proved to be the most robust especially in terms of distinguishing the higher order modes. This section of the code was tested using the DBXTOOL debugging utility to ensure that the  $K$  value the program hones in on does indeed lie between the interval for that mode.

Stage 3 of the program is concerned with solving the ODE and computing the Amplitude Functions  $A(\theta)$  and  $B(\theta)$ . For an exponential density stratified ocean, there exists exact analytical formulae for the functions  $W_1, W_1', W_2, W_2'$ , the solutions of the ODE and the wronskian,  $D$  as follows :

Let

$$\lambda = \sqrt{disc}$$

where

disc	= $4\delta\sigma - \delta^2 - 4k^2$
$\delta$	= $\ln(\text{Density Ratio})$ , the natural logarithm of the ratio between the density of the ocean floor to the ocean surface
$k$	= Wave Number
$\sigma$	= Wave Speed Parameter

If disc  $\geq 0$  then

$$W_1 = e^{\frac{\delta}{2}(1+z)-k} \left[ \left( \frac{2k-\delta}{\lambda} \right) \sin\left(\frac{\lambda(1+z)}{2}\right) + \cos\left(\frac{\lambda(1+z)}{2}\right) \right]$$

$$W_1' = \frac{\delta}{2} W_1 + \frac{e^{\frac{\delta}{2}(1+z)-k}}{2} \left[ (2k-\delta) \cos\left(\frac{\lambda(1+z)}{2}\right) - \lambda \sin\left(\frac{\lambda(1+z)}{2}\right) \right]$$

$$W_2 = e^{\frac{\delta z}{2}} \left[ \left( \frac{\delta-2\sigma}{\lambda} \right) \sin\left(\frac{\lambda z}{2}\right) - \cos\left(\frac{\lambda z}{2}\right) \right]$$

$$W_2' = \frac{\delta}{2} W_2 + \frac{e^{\frac{\delta z}{2}}}{2} \left[ (\delta-2\sigma) \cos\left(\frac{\lambda z}{2}\right) - \lambda \sin\left(\frac{\lambda z}{2}\right) \right]$$

$$D(k, \sigma, h) = (k-\sigma) e^{\delta(\frac{1}{2}-h)-k} \left[ \left( k + \frac{\delta}{2} \right) \frac{\sin\left(\frac{\lambda}{2}\right)}{\frac{\lambda}{2}} + \cos\left(\frac{\lambda}{2}\right) \right]$$

If disc  $< 0$  then Lambda is a complex number so

$$\frac{\sin\left(\frac{\lambda}{2}\right)}{\left(\frac{\lambda}{2}\right)} \rightarrow \frac{\sinh\left(\frac{\lambda}{2}\right)}{\left(\frac{\lambda}{2}\right)}$$

and

$$\cos\left(\frac{\lambda}{2}\right) \rightarrow \cosh\left(\frac{\lambda}{2}\right)$$

where

$$\lambda = \sqrt{-4\mu\sigma + \mu^2 + 4k^2}$$

therefore our set of equations become:

$$W_1 = e^{\frac{\delta}{2}(1+z)-k} \left[ \left( \frac{2k-\delta}{\lambda} \right) \sinh\left(\frac{\lambda(1+z)}{2}\right) + \cosh\left(\frac{\lambda(1+z)}{2}\right) \right]$$

$$W_1' = \frac{\delta}{2} W_1 + \frac{e^{\frac{\delta}{2}(1+z)-k}}{2} \left[ (2k-\delta) \cosh\left(\frac{\lambda(1+z)}{2}\right) - \lambda \sinh\left(\frac{\lambda(1+z)}{2}\right) \right]$$

$$W_2 = e^{\frac{\delta z}{2}} \left[ \left( \frac{\delta-2\sigma}{\lambda} \right) \sinh\left(\frac{\lambda z}{2}\right) - \cosh\left(\frac{\lambda z}{2}\right) \right]$$

$$W_2' = \frac{\delta}{2} W_2 + \frac{e^{\frac{\delta z}{2}}}{2} \left[ (\delta-2\sigma) \cosh\left(\frac{\lambda z}{2}\right) - \lambda \sinh\left(\frac{\lambda z}{2}\right) \right]$$

$$D(k, \sigma, h) = (k-\sigma) e^{\delta(\frac{1}{2}-h)-k} \left[ \left( k + \frac{\delta}{2} \right) \frac{\sinh\left(\frac{\lambda}{2}\right)}{\frac{\lambda}{2}} + \cosh\left(\frac{\lambda}{2}\right) \right]$$

These formulae are contained in the routine "EXPANALYTIC". If the program is run with an exponential density profile the program produces tables which can be used to verify that the analytic and numerical solutions agree. Figure 18 and Figure 19 are tables of data taken from the debugging file which show the numerical and analytic solutions for  $W_0$ ,  $W_1$ ,  $W_1'$ ,  $W_2$ ,  $W_2'$  and the amplitude functions  $A(\theta)$  and  $B(\theta)$ . These tables indicate that the numerical and analytic solutions agree to a satisfactory number of significant figures.

```

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
53
54
55
56
57
58
59
60
61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92
93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108
109
110
111
112
113
114
115
116
117
118
119
120
121
122
123
124
125
126
127
128
129
130
131
132
133
134
135
136
137
138
139
140
141
142
143
144
145
146
147
148
149
150
151
152
153
154
155
156
157
158
159
160
161
162
163
164
165
166
167
168
169
170
171
172
173
174
175
176
177
178
179
180
181
182
183
184
185
186
187
188
189
190
191
192
193
194
195
196
197
198
199
200
201
202
203
204
205
206
207
208
209
210
211
212
213
214
215
216
217
218
219
220
221
222
223
224
225
226
227
228
229
230
231
232
233
234
235
236
237
238
239
240
241
242
243
244
245
246
247
248
249
250
251
252
253
254
255
256
257
258
259
260
261
262
263
264
265
266
267
268
269
270
271
272
273
274
275
276
277
278
279
280
281
282
283
284
285
286
287
288
289
290
291
292
293
294
295
296
297
298
299
300
301
302
303
304
305
306
307
308
309
310
311
312
313
314
315
316
317
318
319
320
321
322
323
324
325
326
327
328
329
330
331
332
333
334
335
336
337
338
339
340
341
342
343
344
345
346
347
348
349
350
351
352
353
354
355
356
357
358
359
360
361
362
363
364
365
366
367
368
369
370
371
372
373
374
375
376
377
378
379
380
381
382
383
384
385
386
387
388
389
390
391
392
393
394
395
396
397
398
399
400
401
402
403
404
405
406
407
408
409
410
411
412
413
414
415
416
417
418
419
420
421
422
423
424
425
426
427
428
429
430
431
432
433
434
435
436
437
438
439
440
441
442
443
444
445
446
447
448
449
450
451
452
453
454
455
456
457
458
459
460
461
462
463
464
465
466
467
468
469
470
471
472
473
474
475
476
477
478
479
480
481
482
483
484
485
486
487
488
489
490
491
492
493
494
495
496
497
498
499
500
501
502
503
504
505
506
507
508
509
510
511
512
513
514
515
516
517
518
519
520
521
522
523
524
525
526
527
528
529
530
531
532
533
534
535
536
537
538
539
540
541
542
543
544
545
546
547
548
549
550
551
552
553
554
555
556
557
558
559
560
561
562
563
564
565
566
567
568
569
570
571
572
573
574
575
576
577
578
579
580
581
582
583
584
585
586
587
588
589
590
591
592
593
594
595
596
597
598
599
600
601
602
603
604
605
606
607
608
609
610
611
612
613
614
615
616
617
618
619
620
621
622
623
624
625
626
627
628
629
630
631
632
633
634
635
636
637
638
639
640
641
642
643
644
645
646
647
648
649
650
651
652
653
654
655
656
657
658
659
660
661
662
663
664
665
666
667
668
669
670
671
672
673
674
675
676
677
678
679
680
681
682
683
684
685
686
687
688
689
690
691
692
693
694
695
696
697
698
699
700
701
702
703
704
705
706
707
708
709
710
711
712
713
714
715
716
717
718
719
720
721
722
723
724
725
726
727
728
729
730
731
732
733
734
735
736
737
738
739
740
741
742
743
744
745
746
747
748
749
750
751
752
753
754
755
756
757
758
759
760
761
762
763
764
765
766
767
768
769
770
771
772
773
774
775
776
777
778
779
780
781
782
783
784
785
786
787
788
789
790
791
792
793
794
795
796
797
798
799
800
801
802
803
804
805
806
807
808
809
810
811
812
813
814
815
816
817
818
819
820
821
822
823
824
825
826
827
828
829
830
831
832
833
834
835
836
837
838
839
840
841
842
843
844
845
846
847
848
849
850
851
852
853
854
855
856
857
858
859
860
861
862
863
864
865
866
867
868
869
870
871
872
873
874
875
876
877
878
879
880
881
882
883
884
885
886
887
888
889
890
891
892
893
894
895
896
897
898
899
900
901
902
903
904
905
906
907
908
909
910
911
912
913
914
915
916
917
918
919
920
921
922
923
924
925
926
927
928
929
930
931
932
933
934
935
936
937
938
939
940
941
942
943
944
945
946
947
948
949
950
951
952
953
954
955
956
957
958
959
960
961
962
963
964
965
966
967
968
969
970
971
972
973
974
975
976
977
978
979
980
981
982
983
984
985
986
987
988
989
990
991
992
993
994
995
996
997
998
999
1000
1001
1002
1003
1004
1005
1006
1007
1008
1009
1010
1011
1012
1013
1014
1015
1016
1017
1018
1019
1020
1021
1022
1023
1024
1025
1026
1027
1028
1029
1030
1031
1032
1033
1034
1035
1036
1037
1038
1039
1040

```

Figure 18 : Numerical Solution

[illegible]

### Figure 18 : Analytic Solution

Stage 4 of the program forms and integrates the three velocity components  $U$ ,  $V$  and  $W$ . To integrate these velocity components using the IMSL routine **DQDAG** requires continuous forms for the functions  $A(\theta)$ ,  $B(\theta)$  and  $K(\theta)$ . However  $A(\theta)$ ,  $B(\theta)$  and  $K(\theta)$  exist only as discrete tabulated functions, therefore some form of interpolation was required.

Three different interpolation techniques were investigated. These were :

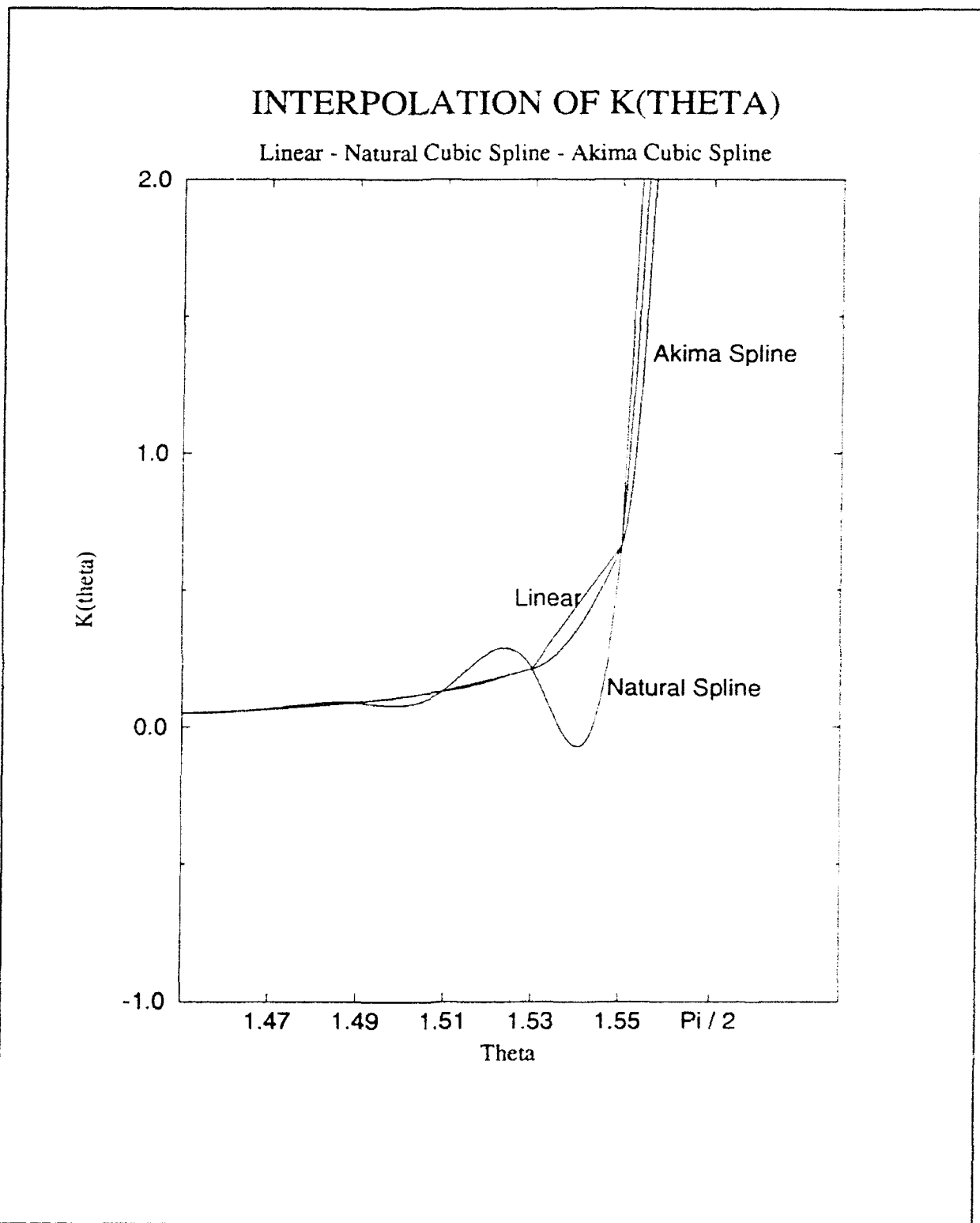
- 1). Linear Interpolation
- 2). Natural Cubic Spline Interpolation
- 3). Akima Cubic Spline Interpolation

Figure 20 shows the three different interpolation methods, used to produce a continuous function for  $K(\theta)$ .

The linear interpolation is quite good and would be satisfactory for  $K(\theta)$ , however for  $A(\theta)$  and  $B(\theta)$ , it was suggested that a smoother interpolation technique would be required.

The next technique investigated was Natural Cubic Spline Interpolation. This technique had some unforeseen side-effects. One of the properties of natural piece-wise cubic splines is that the first derivative at the end of one interval must equal the first derivative at the start of the next interval. however no attempt is made to preserve the function's shape and this can lead to some wild oscillations in certain circumstances as can be seen in the example in Figure 20.

The last technique investigated was Akima Cubic Spline Interpolation (IMSL routine **DCSAKM**). This routine is based on a method by Akima (1970) to combat wiggles in the interpolant. The result is that the shape of the curve produced by **DCSAKM** matches the shape of the data. This, clearly, is what is required and produces the best results of the three interpolation techniques.

Figure 20 : Interpolations of  $K(\theta)$



8. Error Messages

The following is a list of possible error messages

- 1100          Error opening parameter file "WAKE.PAR"**
- The file "WAKE.PAR" must be in the same directory as the executable file
- 1101          Error occurred reading one of the parameters.**
- This means that an error occurred reading the parameter file, possibly one of the parameters comment lines is not exactly 71 characters or the comment line contains tabs
- 1102          Invalid NUMTHETA in Parameter File ( $0 \leq \text{numtheta} \leq \text{maxtheta}$ )**  
**NUMTHETA must be odd.**
- The number of theta values to calculate data for must be within the given range and must be odd.
- 1103          Invalid MODE in Parameter File (  $0 \leq \text{MODE} \leq \text{maxmod}$  ).**
- The mode to calculate data for must be within the given range.
- 1104          Invalid THETAMAX in Parameter File (  $\text{THETAMAX} < \pi/2$  ).**
- The maximum  $\theta$  value allowed must be less than  $\pi/2$ . A value of 1.56 is recommended.
- 1105          Invalid SUBDTH in Parameter File ( $0 \leq \text{SUBDTH} \leq \text{DEPTH}$  ).**
- The depth of the submarine must be within the given range
- 1106          Invalid SUBSPD in Parameter File ( $0 \leq \text{SUBSPD} \leq \text{maxspd}$  ).**
- The speed of the submarine must be within the given range
- 1107          Invalid SUBLEN in Parameter File ( $0 \leq \text{SUBLEN} \leq \text{maxlen}$  ).**
- The length of the submarine must be within the given range
- 1108          Invalid SUBRAD in Parameter File ( $0 \leq \text{SUBRAD} \leq \text{maxrad}$  ).**
- The radius of the submarine must be within the given range

1109 Invalid XEND in Parameter File ( XEND >= XSTART >=0 )

1110 Invalid XSTEP in Parameter File ( XSTEP > 0 ).

1111 Invalid YEND in Parameter File ( YEND >= YSTART >=0 )

1112 Invalid YSTEP in Parameter File ( YSTEP > 0 ).

1113 Invalid ZSTART in Parameter File ( ZSTART must be a multiple of "VALID GRID POINT"

All Z values must lie on the Z Grid as defined by the density profile. The Z Grid is defined by "(((NUMDEN-1)/2)+1)" Z values evenly spaced between 0 and DEPTH

1114 Invalid ZEND in Parameter File ( ZEND must be a multiple of "VALID GRID POINT"

All Z values must lie on the Z Grid as defined by the density profile. The Z Grid is defined by "(((NUMDEN-1)/2)+1)" Z values evenly spaced between 0 and DEPTH

1115 Invalid ZEND in Parameter File ( ZEND >= ZSTART >=0 )

1116 Invalid ZSTEP in Parameter File ( ZSTEP must be a multiple of "VALID GRID POINT"

All Z values must lie on the Z Grid as defined by the density profile. The Z Grid is defined by "(((NUMDEN-1)/2)+1)" Z values evenly spaced between 0 and DEPTH

1117 Invalid ZSTEP in Parameter File ( ZSTEP >=0 )

1118 Invalid DEPTH in Parameter File ( DEPTH > 0)

The depth of the ocean must be greater than zero

1119 Invalid COMPU in Parameter File (COMPU = "Y" or "N").

Compute U Velocity Component, COMPU, must be "Y" for Yes!, or "N" for No!

1120 Invalid COMPV in Parameter File (COMPV = "Y" or "N").

Compute V Velocity Component, COMPV, must be "Y" for Yes!, or "N" for No!

- 1121 Invalid COMPW in Parameter File (COMPW = "Y" or "N").**  
Compute W Velocity Component, COMPW, must be "Y" for Yes!, or "N" for No!
- 1200 Error occurred opening density profile file.**  
Either the file specified by the parameter DENFIL is missing or corrupt
- 1201 Density profile file has a non odd number of points.**  
The density profile must have an odd number of points
- 1202 Density Profile contains "X" points, not "NUMDEN" points as stated in the parameter file.**  
The parameter NUMDEN in the Parameter File should state the correct number of points that are contained in the density profile, DENFIL.
- 1300 Error opening debugging file.**
- 1400 Error opening velocity component file.**
- 1500 Error opening intermediate integration file**
- 1501 Error occurred writing A( $\theta$ ) data to the integration file**  
Either the integration file is missing or it is corrupt
- 1700 Negative Sigma Value passed to Function LOOKUP.**
- 1701 Sigma > Sigarray(MAXMOD) in Function LOOKUP.**  
The sigma value passed to the function lookup is greater than the maximum sigma value allowed, (i.e. the Sigma value for K=0 for the maximum mode calculated (mode 30)).

9. Related Documents

The following documents have been used as references or may be used as further reading.

- SUBMARINE INTERNAL WAVES  
Professor Ernie Tuck, 1992  
Department of Applied Mathematics  
University of Adelaide, S.A.  
Australia
- INTERNAL WAVES RADIATED BY A MOVING SOURCE  
Volume 1. Analytic Simulation  
Michael Milder, 1974  
R and D Associates  
Office of Naval Research  
Advanced Research Projects Agency
- SYNTHETIC APERTURE RADAR IMAGING OF SHIP GENERATED  
INTERNAL WAVES  
Gasparovic, Thompson, Apel 1987  
John Hopkins APL Technical Digest  
Volume 10, Number 4, 1989
- THE DREP INTERNAL WAVE NORMAL MODE MODEL-  
THEORETICAL BACKGROUND  
T.W. Dawson, April 1988  
Technical Memorandum 88-7

REPORT NO.  
MRL-GD-0050AR NO.  
AR-008-257REPORT SECURITY CLASSIFICATION  
Unclassified

## TITLE

The WAKE software suite - a program to predict the internal waves generated by a moving subsurface body in a density stratified ocean: User's guide

AUTHOR(S)  
M.A. CarrollCORPORATE AUTHOR  
DSTO Materials Research Laboratory  
PO Box 50  
Ascot Vale Victoria 3032REPORT DATE  
January, 1993TASK NO  
NAV 92/040SPONSOR  
RANFILE NO  
G6/4/8-4453

REFERENCES

PAGES  
41

CLASSIFICATION/LIMITATION REVIEW DATE

CLASSIFICATION RELEASE AUTHORITY  
Chief, Maritime Operations Division

## SECONDARY DISTRIBUTION

Approved for public release

## ANNOUNCEMENT

Announcement of this report is unlimited

## KEYWORDS

Density Profiles  
Velocity ComponentsComputer Modelling  
2D Wave Plots

Contour Plots

## ABSTRACT

This report provides details of the requirements needed to run the computer program WAKE. WAKE was developed during the period February-March 1992 as a result of a research project which was undertaken within DSTO Salisbury. WAKE calculates the internal wave velocity field generated by a moving subsurface body in a density stratified ocean.

The WAKE Software Suite — A Program to Predict the Internal Waves Generated by a Moving Subsurface  
Body in a Density Stratified Ocean: User's Guide

M.A. Carroll

(MRL-GD-0050)

DISTRIBUTION LIST

Director, MRL  
Chief, Maritime Operations Division  
Research Leader, Airborne Sonar  
M.A. Carroll, EBOR Computing  
MRL Information Service

Chief Defence Scientist (for CDS, FASSP, ASSCM) (1 copy only)  
Director, Surveillance Research Laboratory  
Director (for Library), Aeronautical Research Laboratory  
Director, Electronics Research Laboratory  
Head, Information Centre, Defence Intelligence Organisation  
OIC Technical Reports Centre, Defence Central Library  
Officer in Charge, Document Exchange Centre (8 copies)  
Army Scientific Adviser, Russell Offices  
Air Force Scientific Adviser, Russell Offices  
Navy Scientific Adviser, Russell Offices - data sheet only  
Scientific Adviser, Defence Central  
Director-General Force Development (Land)  
Senior Librarian, Main Library DSTOS  
Librarian, MRL Sydney  
Librarian, H Block  
UK/USA/CAN ABCA Armies Standardisation Republic - DGAT (8 copies)  
Librarian, Australian Defence Force Academy  
Counsellor, Defence Science, Embassy of Australia - data sheet only  
Counsellor, Defence Science, Australian High Commission - data sheet only  
Scientific Adviser to DSTC, C/- Defence Adviser - data sheet only  
Scientific Adviser to MRDC, C/- Defence Attache - data sheet only  
Head of Staff, British Defence Research and Supply Staff (Australia)  
NASA Senior Scientific Representative in Australia  
INSPEC: Acquisitions Section Institution of Electrical Engineers  
Head Librarian, Australian Nuclear Science and Technology Organisation  
Senior Librarian, Hargrave Library, Monash University  
Library - Exchange Desk, National Institute of Standards and Technology, US  
Exchange Section, British Library Document Supply Centre  
Periodicals Recording Section, Science Reference and Information Service, UK  
Library, Chemical Abstracts Reference Service  
Engineering Societies Library, US  
Documents Librarian, The Center for Research Libraries, US

Dr D. Richardson, MOD  
Dr J. Ternan, MOD  
Dr A. Theobald, MOD  
Dr J. Vrbancich, MOD  
Dr J. Smelt, MOD  
Dr R. Webster, MOD  
Dr G. Furnell, MOD  
Chief, Microwave Radar Division, SRL  
Chief, Optoelectronics Division, SRL  
Research Leader, Surveillance Systems, SRL  
Dr G. Haack, MRD, SRL  
Dr C. Anderson, MRD, SRL  
Dr D. Madurasinghe, MRD, SRL  
Dr D. McDonald, MRD, SRL  
Mr W. Cumpston, EBOR Computing